Charge-Mass Equivalence leading to Ilectron from the Electron

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Abstract: Hydrogen atom was considered as the smallest "bit of matter" until the electron was discovered. Nearly all attributes of the electron have been experimentally measured except for its radius. Electron's radius has been derived in classical mechanics. The angular momentum of the electron has been understood as a purely quantum mechanical effect. In this paper, we have established an equivalence between the charge and mass of a fundamental particle. This leads to a definition of a complex charge or a complex mass, which combine both charge and mass. Every fundamental particle with charge and mass can be defined by a single complex charge. Interaction of two complex charges leads to the familiar Coulomb and Gravitational forces. It also points out the possibility of a 5th force of nature. By writing the charge and mass of the electron as mass and charge, we come up with a new particle which we have called the ilectron. Some attributes of the ilectron have been derived in this paper and its relation to Planck's mass and charge are explored. This is a comprehensive paper that has been adapted from material we published in [1-3] for disseminating this information in the Physics community.

Keywords: Electron, Ilectron, Complex Mass, Complex Charge, WIMP

1. Introduction

The electron is a negatively charged massive particle discovered in 1897 by J. J. Thompson [4]. We have listed its properties in Table 1. All these quantities except for r_e have been determined by measurement. The radius is determined by equating the integral of the electric field energy to the kinetic energy ($m_0 c^2$) as follows.

$$m_0 c^2 = \frac{1}{4\pi\varepsilon_0} \int_{r_e}^{\infty} \frac{q^2}{r^2}\, dr = \frac{q^2}{4\pi\varepsilon_0\, r_e} \tag{1}$$

Table 1. Attributes of the electro.

Attribute	Fundamental "Particle"-the electron
rest mass m_0	$9.10938188 \times 10^{-31}$ kg
charge (-q)	$1.602176462 \times 10^{-19}$ C
classical radius r_e	$2.81795518 \times 10^{-15}$ m
quantum mechanical radius	0
angular momentum $\Omega = \hbar / 2$	5.27285×10^{-35} kg m^{2}/s
magnetic moment	$1.001159652\, \mu_B$ μ_B is the Bohr Magneton,$=9.27400899 \times 10^{-24}$ J/T

The result is

$$r_e = \frac{q^2}{4\pi\varepsilon_0 m_0 c^2} \tag{2}$$

with ε_0 =permittivity of free space=8.8541878 $\times$ 10^{-12} F/m. This is the order of magnitude of the size of the electron. It is seen immediately from equation (1) that if the electron had a zero radius, it would have infinite energy. Quantum physicists now regard the electron both as a point particle and a wave, but a point particle conflicts with relativity.

Classically, the electron has to have a nonzero radius, and this implies a structure. To develop a model for a fundamental particle, it is necessary to include not only field energy, but also a mechanical mass (pages 31-32 of [5]). Measurements suggested (page 84 of [6]), and solutions to Dirac's equation (page 116 of [7] and [8] confirmed that the electron has angular momentum. Let us consider a spinning spherical mass *m*, radius *r* spinning about its center, as shown in Figure 1. Such a solid body has a moment of inertia I given by
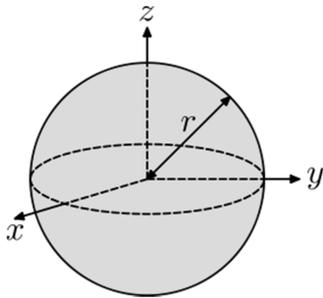
$$I = \frac{2}{5}mr^2 \qquad (3)$$

Figure 1. A solid body of mass m and radius r.

its kinetic energy is

$$E = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{2}{5}m_0 r^2\right)\omega^2 = \frac{1}{5}m_0 r^2\omega^2 \qquad (4)$$

where ω is the angular velocity. The angular momentum is

$$\Omega = I\omega = \frac{2}{5}m_0 r^2\omega \qquad (5)$$

The angular momentum vanishes if the radius goes to zero and a point particle is not acceptable classically. Therefore, the electron spin is described as *"a purely quantum mechanical effect"*.

An obvious interpretation of the solution to Dirac's equation for a stationary electron [9] is that it is moving in a circle of radius $\lambdabar / 2$ where $\lambdabar$ is the reduced Compton wavelength. Using a classically derived equation of motion that introduces the classical stationary state [10] produces a radius some 5% greater, and a velocity some 5% lower, that is consistent with relativity. It is this rotational motion that was termed "*zitterbewgung*" by Schrödinger, the quantum mechanical result being that the velocity was c.

Prior to the development of quantum mechanics, the structure of atoms had been determined by experiment and the results conflicted with classical physics. The application of ideas contributed by Planck, de Broglie, Einstein, Schrödinger, and Heisenberg, culminating in Dirac's equation, laid down the principles of quantum mechanics. In the next section, we briefly consider classical approach to the Hydrogen atom.

This is followed by a discussion of results generated by a complex transformation of the electron leading to speculation about a new particle, in Section 3. We introduce the Ilectron and consider its properties and implications in Sections 4 and 5. We look at the electron as a blackhole and show why it is not a blackhole in Section 6. The Ilectronic mass and charge are compared to the Planck's mass and charge respectively in Sections 7 and 8. In Section 9, we investigate the implication of the equivalence of charge and mass. This leads us to cast Einstein's equation in an equivalent form that shows the relationship between charge and energy. We conclude the paper with summarizing remarks in Section 10 followed by a list of references.

2. Classical Approach to Hydrogen Atom

Hydrogen is the simplest atom we have and was originally considered to be the smallest matter until the electron was discovered. Consider the simple Hydrogen atom illustrated in Figure 2.
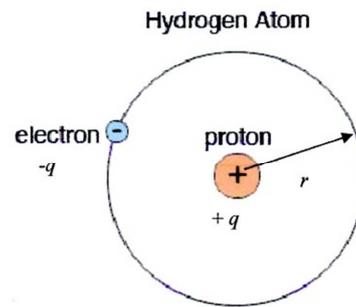

Figure 2. The Hydrogen atom with a single orbiting electron.

The electron experiences a centripetal force and a centripetal acceleration. For any object to stay in a circular motion, there is a centripetal (center-seeking) force, However, as per Newton's laws, there is a reaction force that is centrifugal. This pseudo-force that is centrifugal is balanced by the Coulomb force so that the electron can stay in its circular orbit.

$$\frac{m_o v^2}{r} = \frac{Z q^2}{4\pi \varepsilon_o r^2} \qquad (6)$$

where *v* is the speed of the electron, r the orbit radius, and Z=atomic number (=1 for Hydrogen) and ε_o =permittivity of free space=8.8541878 $\times$ 10^{-12} F/m. Equation (6) can also be written so as to relate the speed and orbit radius as,

$$v = \frac{q}{\sqrt{4\pi\varepsilon_o m_o r}} \quad \text{or} \quad r = \frac{q^2}{4\pi\varepsilon_o m_o v^2} \qquad (7)$$

The result of the orbit radius of equation (7) is consistent with equation (2) from energy considerations. However, classically, an accelerating electron radiates energy (E) at a

rate given by the Larmor formula [11]

$$\frac{dE}{dt} = m_0 \tau \dot{v}^2 \ (J/s) \tag{8}$$

$$\tau = \frac{q^2}{6\pi\varepsilon_0 m_0 c^3} = 6.27 \times 10^{-24} \ (s) \tag{9}$$

The velocity is $\sim \alpha c$ where α is the fine structure constant given by

$$\alpha = \frac{q^2}{4\pi\varepsilon_0 \hbar c} = 7.29735257 \ x \ 10^{-3} \cong \frac{1}{137} \tag{10}$$

and the radius is known as the Bohr radius and is given by

$$a_0 = \frac{4\pi\varepsilon_0 \hbar^2}{m_0 q^2} = 0.529177 \times 10^{-10} \ (m) \tag{11}$$

Writing E_k for the kinetic energy, the lifetime of the orbiting electron is given by

$$\frac{E_k}{\frac{dE}{dt}} = \frac{\frac{1}{2} m_0 v^2}{m_0 \tau \frac{v^4}{r^2}} = \frac{a_0^2}{2\tau\alpha^2 c^2} = 4.67 \times 10^{-11} (s) \tag{12}$$

which is hardly long enough to build a universe!

This problem was overcome by wave mechanics by specifying rules for the motion of electrons in atoms. Dirac's development of his relativistic equation incorporated these rules, but surprisingly produced the result that electrons not acted on by any force, move about at the velocity of light.

3. Complex Electron

In complex variable theory, it is often to link two variables of same dimension. We can do this for the charge and mass of elementary particles, as follows. Considering the electrostatic and the gravitational force between two electrons, we have

$$f_q = \frac{q_1 q_2}{4\pi\varepsilon_0 r^2} = \frac{K q_1 q_2}{r^2} \tag{13}$$

$$f_m = -\frac{m_1 m_2}{4\pi g r^2} = \frac{-G m_1 m_2}{r^2} \tag{14}$$

where, in the usual notation

$$\frac{1}{4\pi\varepsilon_o} = K = 9 \ x \ 10^9 \ (N \, m^2 / C^2) \tag{15}$$

$$\frac{1}{4\pi g} = G = 6.670 \ x \ 10^{-11} \ (N \, m^2 / Kg^2) \tag{16}$$

In the gravitational force above, the convention is a negative force for attraction.

We now define two constants

$$d_1 = \sqrt{\frac{\varepsilon_0}{g}} = 0.861 x 10^{-10} (C/Kg) \tag{17}$$

$$d_2 = \sqrt{\frac{g}{\varepsilon_0}} = 1.16 x 10^{10} (Kg/C) \tag{18}$$

We observe that a dimensionless constant for the electron is given by

$$d_3 = \frac{q}{m_o} d_2 = \frac{q}{m_o} \sqrt{\frac{g}{\varepsilon_o}} = \frac{1.6 x 10^{-19} C}{9.1 x 10^{-31} Kg} d_2 =$$

$$1.75 x 10^{11} d_2 = 2.03 x 10^{21} = \sqrt{\frac{F_{em}}{F_G}} \tag{19}$$

This approach could be extended to all the elementary particles, in particular the proton and the neutron. It is noted that every fundamental particle that has non-zero mass will have an associated constant "d_3". This constant would vanish for a neutral particle *(q=0)* such as a neutron.

We can now express mass and charge with the same dimension by forming a complex Mass (*M*) or a complex charge (*Q*). There are various ways to form these combinations, as follows.

$$M = m + iq d_2 = m + iq \sqrt{\frac{g}{\varepsilon_o}} \tag{20}$$

$$Q = q + im d_1 = q + im \sqrt{\frac{\varepsilon_o}{g}} \tag{21}$$

$$M = \frac{m}{\sqrt{g}} + i \frac{q}{\sqrt{\varepsilon_o}} \tag{22}$$

$$Q = \frac{q}{\sqrt{\varepsilon_o}} + i \frac{m}{\sqrt{g}} \tag{23}$$

For example, if we use the choice of equation (22) and consider complex or combined forces, between two complex masses,

$$M_1 = \frac{m_1}{\sqrt{g}} + i \frac{q_1}{\sqrt{\varepsilon_o}}; \ M_2 = \frac{m_2}{\sqrt{g}} + i \frac{q_2}{\sqrt{\varepsilon_o}} \tag{24}$$

we have the complex force given by

$$F_M = \frac{-M_1 M_2}{4\pi r^2} \tag{25}$$

Expanding this expression using equation (22),

$$F_M = \left\{ -\frac{m_1 m_2}{4\pi g r^2} + \frac{q_1 q_2}{4\pi\varepsilon_0 r^2} \right\} - i \left\{ \frac{m_1 q_2 + m_2 q_1}{4\pi\sqrt{g\varepsilon_0} r^2} \right\} = f_m + i f_{im} \tag{26}$$

Note that the real part gives the well-known gravitational and Coulomb forces. If we now choose M_2 to be a neutral particle by setting

$$q_2 = 0 \qquad F_M^* = f_m - i \left\{ \frac{m_2\, q_1}{4\pi\sqrt{g\,\varepsilon_0}\, r^2} \right\} \qquad (27)$$

Setting

$$q_1 = m_{im}\sqrt{\frac{\varepsilon_0}{g}} \qquad (28)$$

we obtain

$$F_M^* = f_m + i\, f_{im} \quad f_{im} = -\frac{m_2 m_{im}}{4\pi\, g\, r^2} \qquad (29)$$

The strength of a force is given by Matt Strassler [12, 13]

$$S = \frac{r^2 f}{\hbar c} \qquad (30)$$

Comparing the strength of this imaginary force to the electric force, and assuming the neutral particle is a neutron

$$S_{im/e} = \frac{m_n}{q}\sqrt{\frac{\varepsilon_0}{g}} \sim 9.00 x\, 10^{-19} \qquad (31)$$

The strengths of the four standard forces together with this new force are given below in Table 2. These strengths are calculated in the regime where they are effective, the strong force within the nucleus, and the weak force within a nucleon while the electromagnetic force operates outside the electron.

Table 2. *Strengths of the five forces.*

Strong	Weak	Electro Magnetic	Imaginary Electro-Mass	Gravity
0.11	0.020	0.007 (=α)	6.6×10^{-21}	2.6×10^{-35}

A force is considered weak if $F\, r^2 \ll (\hbar c)$ and is considered strong if $F\, r^2 \approx (\hbar c)$. In particle physics the electro-mass force would be undetectable, but if the neutral 'particle' is a neutron star, this force would be the dominant force acting on electrons. A typical neutron star contains $\sim 25 \times 10^{56}$ neutrons.

4. The Ilectron

Recognizing that imaginary mass can be interpreted as charge, and imaginary charge as mass, we can consider the *imaginary electron,* where we interchange the mass and charge, but this transformation is not merely an algebraic transform. We must transform the intrinsic mass of the electron to obtain the new charge, the charge transforming as before.

$$i[m_i, q] = [im_i, iq] = \left[\sqrt{\frac{g}{\varepsilon_0}}\, q, \sqrt{\frac{\varepsilon_0}{g}}\, m_i \right] = [m_{ii}, q_i] \qquad (32)$$

Evaluating the components

$$m_{ii} = q_e\sqrt{\frac{g}{\varepsilon_0}} = 1.86 \times 10^{-9}\,\text{kg} \qquad \text{and} \qquad (33)$$

$$q_i = m_i\sqrt{\frac{\varepsilon_o}{g}} = \frac{m_0}{3.2635}\sqrt{\frac{\varepsilon_0}{g}} = 2.17 \times 10^{-41}\,\text{C} \qquad (34)$$

In equation (33), m_{ii} is the intrinsic mass equivalent of the electron's charge and q_i is the charge equivalent of electron's intrinsic mass. We are calling this new particle with its mass and charge given by equation (34) as the "*ilectron*" or an "*imaginary electron*".

5. Ilectron Spin and Magnetic Moment

If we assume the spin of $\hbar/2$ is transferred to the new mass we can determine the magnetic moment. The fine structure constant is given by

$$\alpha = \frac{q^2}{4\pi\varepsilon_0 \hbar c} \qquad (35)$$

Replacing q by q_i

$$\alpha_i = \frac{q_i^2}{4\pi\varepsilon_0 \hbar c} = \left(\frac{q_i}{q}\right)^2 \alpha = \frac{7.85^2 \times 10^{-82}}{1.602^2 \times 10^{-38}} = 2.401 \times 10^{-43} \qquad (36)$$

The rotational velocity is

$$\frac{v_0}{c} = \eta_e = \frac{\sqrt{1+\frac{3}{\alpha_i}} - 1}{\sqrt{\frac{3}{\alpha_i}}} = \sqrt{1 + \frac{\alpha_i}{3}} - \sqrt{\frac{\alpha_i}{3}} = 1 - 2.829 \times 10^{-22} \qquad (37)$$

$$r = \frac{v_0 \tau_{ii}}{\left[1 - \frac{v_0^2}{c^2}\right]^{3/2}} \qquad (38)$$

where

$$\tau_{ii} = \frac{q_i^2}{6\pi\varepsilon_0 m_i c^3} = \frac{2}{3}\frac{\alpha_i \hbar}{m_{ii} c^2} = \frac{2}{3}\left(\frac{q_i}{q}\right)^2 \frac{\alpha \hbar}{c} = \frac{2}{3}\left(\frac{q_i}{q}\right)^2 \frac{\lambda}{c} \qquad (39)$$

and λ is the reduced Compton radius. The radius of rotation is then

$$r = \frac{2}{3}\left(\frac{q_i}{q}\right)^2 \frac{\lambda}{c} \frac{v_0}{\left[1 - \frac{v_0^2}{c^2}\right]^{3/2}}$$

$$= \frac{2}{3}\frac{2.401 \times 10^{-43}}{7.297 \times 10^{-3}}\frac{3.861 \times 10^{-13}}{13.46 \times 10^{-33}} = 6.3 \times 10^{-18} m \qquad (40)$$

The magnitude of the magnetic moment is then given by

$$\mu_m = q_i v_0 r = 1.36 \times 10^{-22} qc \times \frac{6.3 \times 10^{-18}}{9.27 \times 10^{-24}} \mu_B$$

$$= 1.48 \times 10^{-27} \mu_B \qquad (41)$$

The above ideas may be considered as mere speculation, but where would science be if some of us did not occasionally break away from what is known.

6. Electrons and Black Holes

Briefly, it is worthwhile to look at some analogous relationship between the electron and a black hole.

The classical electron radius is given by the Lorentz formula [5] (also equations (2) and (7) above),

$$r_0 = \frac{q_e^2}{4\pi\varepsilon_0 m_0 c^2} = 2.817 \; x \; 10^{-15} \, m \qquad (42)$$

Replacing $q_e / \sqrt{\varepsilon_0}$ by $m_0 / \sqrt{g}$ in the above, the radius of the electron is given by

$$r_0 = \frac{m_0}{4\pi g c^2} \qquad (43)$$

A black hole is a region of space and time with a strong gravitational pull so that no particle or EM radiation can escape from it. The boundary from which no escape is possible is called the "event horizon". The above formula for the radius of an electron is similar to the formula for the radius of a Schwarzschild black hole

$$r_S = \frac{m}{2\pi \; g \; c^2} = \frac{2 \, G \, m}{c^2} \qquad (44)$$

Examples of Schwarzschild radii of some common planets are: sun (3 km), earth (8.7 mm), Moon (0.11 mm) and Jupiter (2.2m). If the Schwarzschild radius exceeds the physical radius, the object is a black hole. Hence these planets are not black holes. Alternatively, we estimate the Schwarzschild radius for an electron as

$$r_{es} = \frac{m_e}{4\pi \; g \; c^2} = 6.76 \times 10^{-58} m \qquad (45)$$

whereas the classical radius of the electron is 2.82 x 10^{-15} m. With the classical radius of an electron being much larger than its Schwarzschild radius, it is not a black hole.

By forming a complex electron, we have raised the possibility of a new force many orders weaker than the electromagnetic force, yet still fourteen orders greater than gravity. Continuing with this idea we constructed a hypothetical imaginary electron which turns out to have a substantial mass of $\sim 1.9 \times 10^{-9}$ kg and effectively no charge or magnetic moment.

The ilectron would appear to be a good contender for a WIMP (a weakly interacting massive particle), so far undetected hypothetical particle in one explanation of dark matter. If the ilectron exists, presumably it has an anti-particle and an ilectron meeting an anti-ilectron would result in a pair of high energy gamma photons. Assuming a similar imbalance in the relative numbers as is found for other particle pairs, such interactions would be rare. The detection of these gamma rays would in all probability require the design of new detectors. The ilectron has no impact on positron. The anti-particle to the ilectron is the ipositron.

The mass of the ilectron is 1.86 x10^{-9} kg which converts to $\sim 10^{21}$ MeV=10^{15} TeV. The maximum energy of the LHC in CERN is 13TeV. The LHC would have to be upgraded by a factor of around 8x10^{13} to produce an ilectron!

Currently the only possibility of confirmation is if the ilectron meets its anti-particle, the ipositron and the resulting gammas pass through our detectors. Gammas above 100 TeV are classified as ultra-high energy, and so far none have been detected.

If they are the elusive WIMP, they would have to provide a mass of $\sim$90% of the Milky Way. The mass of the sun is $\sim$2x10^{30} kg and so the number of ilectrons required is

$$N=[2x10^{30}\text{kg}/1.86 \; x \; 10^{-9}] \sim 10^{39} \qquad (46)$$

With the volume of the Milky Way being $\sim 10^{48}$ m^3 the required density is

$$\rho_N = 10^{-8} \; m^{-3} \qquad (47)$$

Detection of gammas of this energy would not only support their existence but would also enable estimates of the density of ilectrons. Scattering of these gammas with electrons may also contribute to an explanation of gamma ray bursts.

From classical theory, the electron has a radius which is larger than its Schwarzschild radius, and hence, is not a black hole. This observation of the similarity of the radius of the electron [equation (38)] and the radius of a black hole [equation (39)] raises an interesting question-if the charges of particles are quantized, are the masses of black holes quantized? That is a question for Astrophysicists and may already have been addressed. The known properties of the electron and the derived properties of the ilectron are considered in Table 3. The classical radius of the electron is given in Table 3.

In Table 3, we have assumed the angular momentum of the ilectron to be the same as for the electron. The spin velocity for the ilectron being so close to c means that the spin radius is remarkably close to the maximum value.

Table 3. *Comparison of the attributes of the electron and the ilectron.*

Attribute	Fundamental "Particle"-the electron	Hypothesized "Particle"-the ilectron
rest mass	m_{oE}=9.10938188$\times 10^{-31}$ kg	$m_{oi} = q_E \sqrt{\dfrac{g}{\varepsilon_o}}$ =1.86 x 10^{-9} kg=1.86 µg (Very heavy !)

Attribute	Fundamental "Particle"-the electron	Hypothesized "Particle"-the ilectron
charge	q_E=1.602176462$\times 10^{-19}$ C	$q_I = m_{iE}\sqrt{\dfrac{\varepsilon_0}{g}} = \dfrac{m_{0E}}{3.2635}\sqrt{\dfrac{\varepsilon_0}{g}}$ =2.17 x 10^{-41} C with m_{iE} as the electron's intrinsic mass
classical radius	r_E=2.81795518$\times 10^{-15}$ m	Not calculated
quantum mechanical radius	0	Assumed zero
angular momentum	$\Omega = \hbar/2$ =5.27285$\times 10^{-35}$ kg m$^{2/}$s	$\Omega = \hbar/2$ =5.27285$\times 10^{-35}$ kg m$^{2/}$s (an assumption)
Magnetic moment	1.001159652 m_B m_B is the Bohr Magneton,=9.27400899$\times 10^{-24}$ J/T	1.48 x 10^{-27} m_B based on the assumption above μ_B is the Bohr Magneton,=9.27400899$\times 10^{-24}$ J/T
Fine structure constant FSC	$\alpha_E = \dfrac{q_E^2}{4\pi\varepsilon_0\hbar c}$ =7.29735x10^{-3} $\simeq$(1/137)	$\alpha_I = \dfrac{q_I^2}{4\pi\varepsilon_0\hbar c}$ =1.3387 x 10^{-46}
Spin velocity	$\dfrac{v_{sE}}{c} = \dfrac{\sqrt{1+\dfrac{3}{\alpha_E}}-1}{\sqrt{\dfrac{3}{\alpha_E}}}$ 0.9518967	$\dfrac{v_{sI}}{c} = \dfrac{\sqrt{1+\dfrac{3}{\alpha_I}}-1}{\sqrt{\dfrac{3}{\alpha_I}}}$.=1-6.68 x 10^{-24} almost 1, but not quite!

Designating radius as r_∞ it can be shown that

$$r_{sE} = \frac{\lambdabar}{2\sigma} = 2.02838\times 10^{-13}$$

$$r_{sI} \sim r_\infty = \frac{r_{sE}}{\sigma} = \frac{\lambdabar}{2\sigma^2} = 2.13087\times 10^{-13}\,m \qquad (48)$$

where

$$\sigma = \frac{v_{sE}}{c} \qquad (49)$$

Spin of the electron is a quantum mechanical property and is an intrinsic form of angular momentum. Some physicists think of this as the earth rotating on its own axis in 24 hours-a spinning top. This view, however, is not mathematically justifiable.

In Table 4, we list some fundamental physical constants.

Table 4. Some fundamental constants.

Physical Constant	Notation	Value and Units
Planck's constant	h	6.62607 x 10^{-34} m^2 kg/s
Planck's charge	$q_P = \sqrt{2\varepsilon_0 h c}$	1.8755 x 10^{-18} C
Planck's Mass	$m_P = \sqrt{\dfrac{h c}{2\pi G}}$	21.764 μg Or, in energy units 1.220910$\times 10^{19}$ G eV
Planck's length	l_P	1.6 x 10^{-35} m
Speed of light in vacuum	c	3 x 10^8 m/s
Permittivity of free space	ε_0	8.85418 x 10^{-12} F/m
Gravitational constant	G	6.67030 x 10^{-13} m^3 kg^{-1} s^{-2}
Constant related to G	$g = \dfrac{1}{4\pi G}$ This is not acceleration of earth's gravity	1.192 x 10^9 m^{-3} kg s^2

7. Ilectronic Mass and Its Relation to Planck's Mass

7.1. Planck's Mass

Denoted by m_P, is the unit of mass in the system of natural units known as Planck units [14]. It is approximately 0.02 milligrams. Unlike some other Planck units, such as Planck length, Planck mass is not a fundamental lower or upper bound; instead, Planck mass is a unit of mass defined using only what Planck considered fundamental and universal units.

It is defined as

$$m_P = \sqrt{\frac{h c}{2\pi G}} \qquad (50)$$

where h is the Planck's constant=6.6260 x 10^{-34} m^2 kg/s and G is the gravitational constant=$\dfrac{1}{4\pi g}$ =6.670 x 10^{-11} (Nm^2/kg^2) or m^3 kg^{-1} s^{-2}

resulting in g=1.192 x 10^9 m^{-3} kg s^2

Substituting these physical constants, we get

$$m_P=21.764 \text{ μg} \qquad (51)$$

Using the mass-energy equivalence E=m c^2, Planck mass converts to

$$m_P = 1.220910 \times 10^{19} \text{ GeV} \quad (52)$$

In comparison, this value is of the order of 10^{15} (a quadrillion) times larger than the highest energy available (13 TeV) in the Large Hadron Collider at CERN.

7.2. Ilectron Mass

We had earlier derived the ilectron mass as

$$m_{ii} = q_E \sqrt{\frac{g}{\varepsilon_o}} \quad (53)$$

Rewriting the Planck's mass from equation (50)

$$m_P = \sqrt{\frac{hc}{2\pi G}} \quad (54)$$

Taking the ratio of two masses, we have

$$\frac{m_P}{m_{ii}} = \frac{\text{Planck's mass}}{\text{Ilectron mass}} = \frac{\sqrt{2hc\varepsilon_o}}{q_E} = \frac{1}{\sqrt{\alpha_E}} \simeq 11.706 \quad (55)$$

We recognize that the ratio of Planck's mass to ilectron mass is closely related to the fine structure constant of the electron α_E.

In terms of the actual masses, we have

$$m_{ii} = q_e \sqrt{\frac{g}{\varepsilon_o}} = 1.86 \times 10^{-9} \text{ kg} = 1.86 \ \mu g \sim 10^{21} \text{ MeV} = 10^{15} \text{ TeV} \quad (56)$$

$$\text{So} \ \frac{m_P}{m_{ii}} = \frac{\text{Planck's mass}}{\text{Ilectron mass}} = \frac{21.764 \ \mu g}{1.86 \ \mu g} = 11.70 \quad (57)$$

which is consistent with equation (50).

8. Ilectronic Charge and Its Relation to Planck's Charge

Planck's charge is given by [15]

$$q_P = \sqrt{4\pi\varepsilon_o (h/2\pi)c} = \sqrt{2\varepsilon_o hc} \quad (58)$$

Ilectronic charge is given by

$$q_i = m_{iE} \sqrt{\frac{\varepsilon_o}{g}} = \frac{m_{oE}}{3.2635} \sqrt{\frac{\varepsilon_o}{g}} \quad (59)$$

with m_{iE} is the electron's intrinsic mass and m_{oE} is the rest mass of the electron. We also have the Fine structure constant of the ilectron given by

$$\alpha_i = \frac{q_i^2}{4\pi\varepsilon_o hc} = \frac{q_i^2}{q_P^2} \quad (60)$$

$$\frac{q_i}{q_P} = \sqrt{\alpha_i} = \sqrt{1.3387 x 10^{-46}} = 1.157 x 10^{-23} \quad (61)$$

We can also get this ratio in another way

$$\frac{\text{ilectron charge}}{\text{Planck's charge}} = \frac{q_i}{q_P} = \frac{2.17 x 10^{-41} \text{C}}{1.8755 \ x 10^{-18} \text{C}} = 1.157 \times 10^{-23} \quad (62)$$

Which is consistent with equation (61)
We can write the reciprocal of this ratio as

$$\frac{\text{Planck's charge}}{\text{ilectron charge}} = \frac{q_P}{q_i} = \frac{1.8755 x 10^{-18} \text{C}}{2.17 \ x 10^{-41} \text{C}} = 8.642 \times 10^{22} \quad (63)$$

9. Mass and Energy Equivalence Leading to Charge and Energy Equivalence

Sir Isaac Newton believed mass and energy are two distinct and unrelatable quantities. In the Newtonian scheme, mass is a measure of inertia or quantity of matter that resists its motion, and mass and energy have distinct and separate identities. However, Einstein's most famous equation

$$E = mc^2 \quad (64)$$

explicitly states that mass and energy are interconvertible. Mass does not have to move to have energy. Just need to have mass. This simple equation has enjoyed profound consequences. Now that we have found an "equivalence" between mass and charge, we can rewrite equation (59) as

$$E = q \sqrt{\frac{g}{\varepsilon_o}} c^2 = q \ d^2 \quad (65)$$

where

$$d = \sqrt{\sqrt{\frac{g}{\varepsilon_o}}} \ c = 1.0777 \times 10^5 \ c = 3.2333 \times 10^{13} \ (\sqrt{Voltage}) \quad (66)$$

Furthermore, dimensionally speaking, Energy=Charge x Voltage, which suggests, the dimension of the constant quantity d in equation (60) is the square root of Voltage, and this has been verified. An alternate view is to regard equation (60) as defining charge having an effective mass of

$$M_q = q \sqrt{\frac{g}{\varepsilon_0}} \quad (67)$$

Equation (62) indicates that charge and energy are two sides of the same coin.

10. Summarizing Remarks

We have found that:
Mass of the ilectron and Planck's mass are simply related by the fine structure constant of the electron.

$$\frac{m_P}{m_{ii}} = \frac{\text{Planck's mass}}{\text{Ilectron mass}} = \frac{1}{\sqrt{\alpha_E}} \simeq 11.706 \quad (68)$$

Charge of the ilectron and Planck's charge are simply related by the fine structure constant of the ilectron.

$$\frac{\text{Planck's charge}}{\text{ilectron charge}} = \frac{q_P}{q_i} = \frac{1}{\sqrt{\alpha_i}} = 8.642 \times 10^{22} \qquad (69)$$

We have established an equivalence between mass and charge, and this leads us to the following observation. Einstein's equation firmly established the interconvertibility of mass and energy with profound consequences. This leads us to the following

$$E = q \ d^2 \text{ similar to } E = m \ c^2 \qquad (70)$$

where E is energy, m is mass, c=speed of light in vacuum, q is charge and d is a physical constant simply related to the speed of light, as can be seen in equation (61). The consequences of the interconvertibility of charge and energy are yet to be determined.

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