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# Quantum Zeno Effect and Atomic Population Inversion

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**Abstract:** Quantum Zeno effect can be applied to quantum information processing, and can reveal the nature of quantum measurement. In addition, it has also many potential applications. This suggests that studying the quantum Zeno effect has great theoretical and experimental significance. In this work, the system of a two-level atom interacting with a single mode field is considered and the dynamics of the system subjected to successive projection measurements is studied, and the quantum Zeno effect is presented. Moreover, the influence of the quantum Zeno effect on atomic population inversion is discussed. Based on Schrödinger equation, the survival probability of the initial state of the two-level atom subjected to frequently repeated measurements can be obtained. The survival probability depends on the time interval between measurements. It is seen that the exponential decay of the system under slowly frequent measurements is presented instead of the naturally oscillatory process. For slowly repeated measurements the atomic population inversion and the survival probability of initial state decline rapidly at the early time and then both of them become unchanged. As the time intervals of the measurements are sufficiently short, the quantum Zeno effect occurs. These results have also shown how the measurement can inhibit the atomic population inversion.

**Keywords:** Quantum Zeno Effect, Population Inversion, JC Model

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## 1. Introduction

The development of quantum information and quantum computing over the last two decades has led to increasing attention to quantum Zeno effect (QZE) in the literature [1-4]. The QZE is firstly proposed by Misra and Sudarshan [5], and Itano, Heinzen, and Wineland have observed the effect through experiment in 1990 [6]. The QZE is a quantum effect that unstable quantum systems are induced to the reduction of the quantum decay rate even stop due to the frequent measurements at short time intervals [5-10]. An opposite effect known as quantum anti-Zeno effect (QAZE), i.e., the acceleration of the decay process caused by frequent measurement, has also been pointed out. In theory, after the frequent projection measurements for a two-level atoms, Francica et al. have demonstrated how both quantum and classical correlations are affected by the QZE and QAZE [1]. Moreover, Chaudhry and Gong have revealed the QZE and QAZE in the model of dephasing [2]. Interestingly, Porras has found that the QZE can make free-moving particles to stay in specific spatial areas [3]. In addition, David and Martín-Martínez et al. used the QZE to perform

non-projective measurements in finite frequency to obtain the quasi-Zeno effect [4]. In the meantime, the QZE has obtained wide attention in experiments. For example, Erez and Gordon et al. concluded that for frequent quantum measurements of a quantum system its entropy and temperature differ from the rate of change described in classical thermodynamic criteria [11]. Unlike previous most of the research is on the basis of the interaction with atoms, Peise and Lücke et al. used laser beams to realize no interaction measurements, resulting in the indirect QZE, thus inhibiting decay of ultracold atoms [12]. Because of these theories and experiments, the QZE has been widely used in various quantum protocols. Such as, the QZE can protect quantum entanglement [13], and Zheng and Xu et al. demonstrated by experiments this conclusion in the nuclear magnetic resonance system [14]. At the same time, the QZE has also been used in the preparation of controllable entanglement [15], all-optical exchange [16] and optical modulation [17].

It is well known that population inversion is an important quantum feature. In recent years, the research on the population inversion has made a great breakthrough [18-22]. However, very few works addressed the issue of the impact of population inversion by QZE. In this paper, the influence of

different measurement time intervals on the time evolution of the transition and initial survival probability is studied in the system of atom-field interaction of JC model [23]. It is shown that when the measured time interval is short enough, the QZE will be generated, and the influence of QZE on the population inversion is investigated.

## 2. Model and Hamiltonian

Considering a two-level atom in the single mode field, the Hamiltonian of the total system in the rotating-wave approximation is ( $\hbar=1$ )

$$H=H_0+H_1 \tag{1}$$

where,

$$H_0 = \frac{1}{2} \omega \sigma_z + \Omega a^\dagger a \tag{2}$$

and

$$H_1 = g (\sigma_- a^\dagger + \sigma_+ a) \tag{3}$$

$H_0$  represents the Hamiltonian of single-mode field and two-level atom, and  $H_1$  is the interaction between the field and the atom;  $a^\dagger$  and  $a$  are the creation and annihilation operators of the field;  $\omega$  and  $\Omega$  are the transition frequency of the atom and the frequency of the field;  $g$  is the coupling strength between the atom and the field;  $\sigma_z$ ,  $\sigma_+$  and  $\sigma_-$

are the Pauli operator, creation operator and annihilation operator of the atom, respectively.

In the interaction picture, the Hamiltonian can be written as

$$V = g(\sigma_- a^\dagger e^{-i\Delta t} + \sigma_+ a e^{i\Delta t}) \tag{4}$$

where,  $\Delta = \omega - \Omega$ . The interaction energy (4) only can cause transitions between the states  $|a, n\rangle$  and  $|b, n+1\rangle$  ( $|a\rangle$  and  $|b\rangle$  are the excited state and the ground state). The state vector of the system is therefore represented as

$$|\Psi(t)\rangle = \sum_n C_{a,n}(t) |a, n\rangle + C_{b,n+1}(t) |b, n+1\rangle \tag{5}$$

The equations of motion for the probability amplitudes  $C_{a,n}(t)$  and  $C_{b,n+1}(t)$  are obtained by first substituting for  $|\Psi(t)\rangle$  and  $V$  from (5) and (4) in Schrödinger equation ( $i \frac{\partial}{\partial t} |\Psi(t)\rangle = V |\Psi(t)\rangle$ ) and then projecting the resulting equations onto  $\langle a, n|$  and  $\langle b, n+1|$ , respectively. Then it is obtained

$$\dot{C}_{a,n} = -ig\sqrt{n+1}e^{i\Delta t}C_{b,n+1} \tag{6}$$

$$\dot{C}_{b,n+1} = -ig\sqrt{n+1}e^{-i\Delta t}C_{a,n} \tag{7}$$

A general solution for the probability amplitudes is

$$C_{a,n}(t) = \left\{ C_{a,n}(0) \left[ \cos\left(\frac{\Omega_n}{2}t\right) - \frac{i\Delta}{\Omega_n} \sin\left(\frac{\Omega_n}{2}t\right) \right] - C_{b,n+1}(0) \frac{2ig\sqrt{n+1}}{\Omega_n} \sin\left(\frac{\Omega_n}{2}t\right) \right\} e^{i\Delta t/2} \tag{8}$$

$$C_{b,n+1}(t) = \left\{ C_{b,n+1}(0) \left[ \cos\left(\frac{\Omega_n}{2}t\right) + \frac{i\Delta}{\Omega_n} \sin\left(\frac{\Omega_n}{2}t\right) \right] - C_{a,n}(0) \frac{2ig\sqrt{n+1}}{\Omega_n} \sin\left(\frac{\Omega_n}{2}t\right) \right\} e^{-i\Delta t/2} \tag{9}$$

where,  $\Omega_n^2 = \Delta^2 + 4g^2(n+1)$ . All the physically relevant quantities relating to the quantized field and the atom can be obtained from these equations.

## 3. The Role of Quantum Measurement in Atomic Population Inversion

Atomic population inversion means the difference of probability between the ground state and excited state of the atom. Considering the quantum-dynamical process in the absence of repeated measurements, the atomic population inversion can be written as

$$W(t) = \sum_n (|C_{a,n}(t)|^2 - |C_{b,n}(t)|^2) \tag{10}$$

The probability of the system being in the initial state  $|\Psi(0)\rangle$  at later times is

$$S(t) = |\langle \Psi(0) | \Psi(t) \rangle|^2 \tag{11}$$

Next it would be interesting to investigate the dynamics of the system subjected to successive measurements. It is assumed that the initial state of the system is  $|\Psi(0)\rangle = |a, 0\rangle$  and the instantaneous ideal projection measurements are performed at regular time intervals  $\tau$ . When without repeated measurements are done, the probability of the system being in the initial state at time  $t$  is

$$S(t) = |\langle a, 0 | \Psi(t) \rangle|^2 = |C_{a,0}(t)|^2 \tag{12}$$

After  $N$  measurements have been performed at time  $t = N\tau$ , the survival probability of the initial state is

$$S^{(N)}(t) = [S(\tau)]^N = |\cos^2 g\tau|^N \tag{13}$$

In the short-time approximation, when the interval time of the measurement is enough short, the equation (13) can be written as

$$S^{(N)}(t) = \exp[-\gamma(\tau)t] \tag{14}$$

where the effective decay rate  $\gamma(\tau) = g^2\tau$ . After a series of measurements, the population inversion is

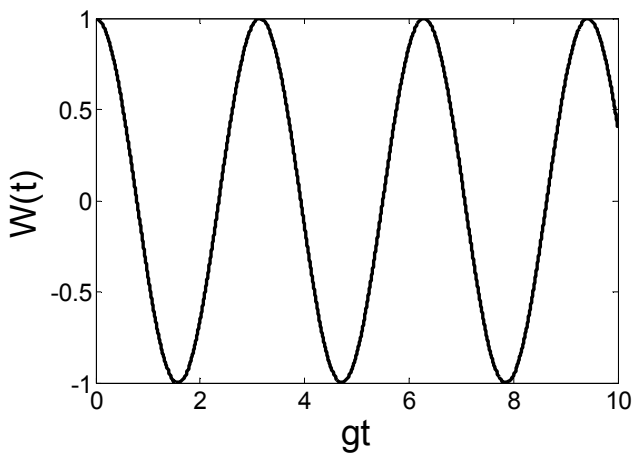
$$W^{(N)}(t) = 2 \exp[-\gamma(\tau)t] - 1 \tag{15}$$

In the following figures analysis, the dimensionless time  $gt$  and  $g\tau$  will be chosen as time variables and time intervals of measurement, respectively.

### 4. Numerical Analysis

Based on the above theoretical calculations, the measurement of influence population inversion will be analyzed. Moreover, it is easily seen how the QZE influences the population inversion and the survival rate of the initial state.

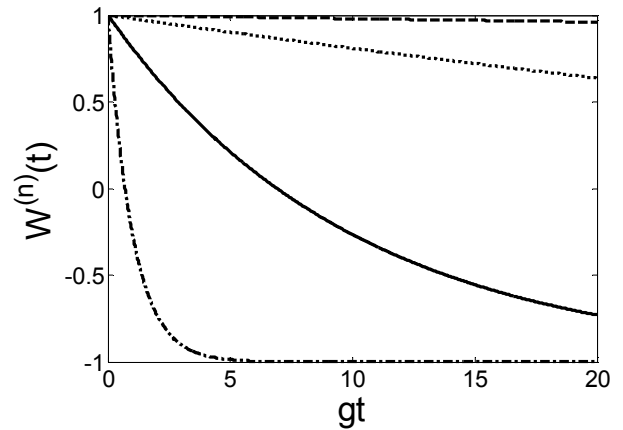
As a comparison, the evolution curve of atomic population inversion is shown in figure 1, while the system without repeated measurements are done. Here, only vacuum field is considered as the initial field state ( $n = 0$ ). It can be clearly seen from figure 1 that the atomic population inversion is always followed with Rabi oscillation, which has the characteristics of periodic recovery and collapse, and that is a well-known result.



**Figure 1.** The change of population inversion  $W(t)$  with time  $gt$  in the case of no repeated measurements for the system. The field is initially in vacuum state and the atom in the excited state.

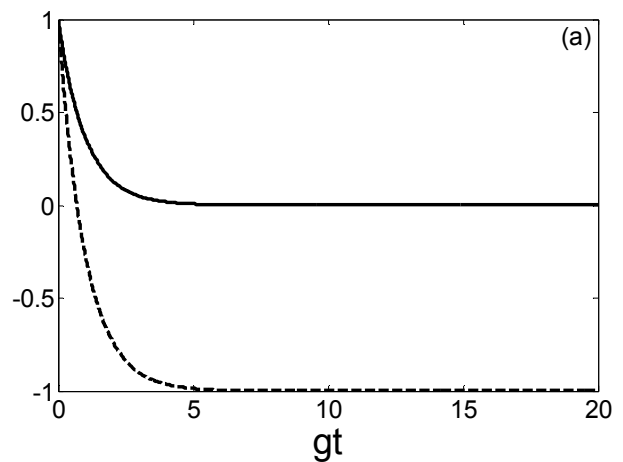
Figure 2 is the evolution curve of atomic population inversion when different values for the measured time intervals  $\tau$  are taken in the case of measurements of the system. From the figure it can be found that when the time intervals between measurements are larger, such as  $g\tau = 1$  (dash-dot line), the atomic population inversion is without showing oscillations and has a distinct decay, but the QZE is not found. Therefore, measurement can effectively inhibit population inversion. Compared with  $g\tau = 1$  (dash-dot line),

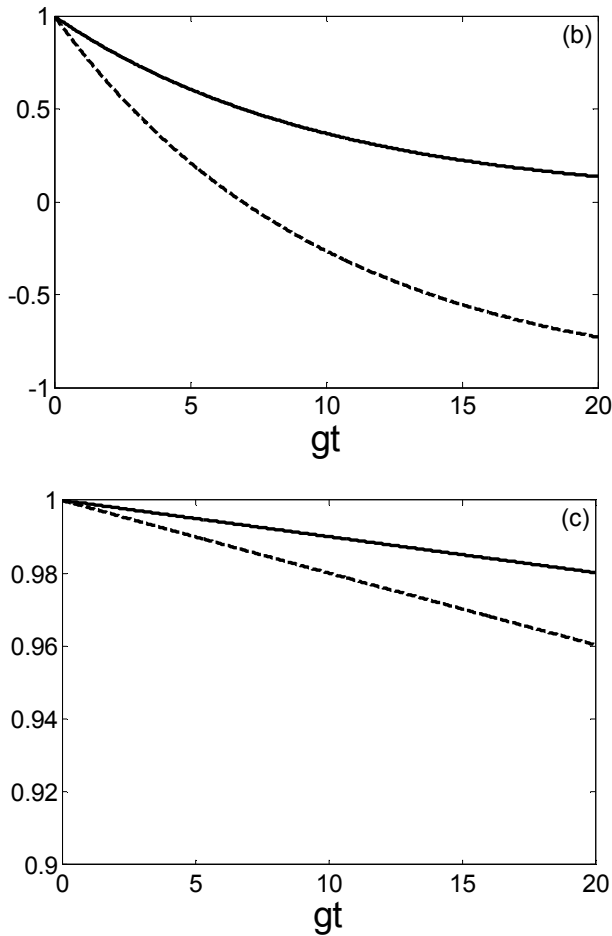
the rate of decay of atomic population inversion slows down when measured time intervals become short, such as  $g\tau = 0.1$  (solid line),  $g\tau = 0.01$  (dotted line), and  $g\tau = 0.001$  (dashed line). However, for  $g\tau = 0.1$  (solid line) and  $g\tau = 0.01$  (dotted line), the QZE still do not occur. However, when the measured time intervals are short enough, the QZE is observed (shown in figure 2, dashed line).



**Figure 2.** The change of population inversion  $W^{(N)}(t)$  with the time  $gt$  in the case of repeated measurements for the system and different time-intervals. The field is initially in vacuum state and the atom in the excited state. ( $g\tau = 1$ , dash-dot line;  $g\tau = 0.1$ , solid line;  $g\tau = 0.01$ , dotted line;  $g\tau = 0.001$ , dashed line).

From the Figure 3(a), it can be seen that the atomic population inversion (dashed line) and survival probability (solid line) of initial state rapid decline for  $g\tau = 1$  as  $gt \in [0, 5]$ , then both of them are no longer changed as  $gt > 5$ . Compared with figure 3(a), it can be seen from figures 3(b) and 3(c) that the curve of atomic population inversion and survival probability of initial state becomes more and more closed when the measured time intervals become short. Specially in figure 3(c), frequent measurement of the system results in the QZE. In addition, it can be concluded that the atomic decay rate and survival probability of initial state are close to unit as  $g\tau$  decrease further.





**Figure 3.** The changes of population inversion  $W^{(N)}(t)$  (dashed line) and survival probability  $S^{(N)}(t)$  (solid line) with time  $gt$  in the case of measurements for the system and different time-intervals. The field is initially in vacuum state and the atom in the excited state. (a)  $g\tau=1$ ; (b)  $g\tau=0.1$ ; (c)  $g\tau=0.001$ .

## 5. Conclusion

By using the JC model of interaction between atom and radiation fields, the influence of the QZE on atomic population inversion was investigated. It has been shown that when no measurements are done on the system, the atomic population inversion shows Rabi oscillations, which has the characteristics of periodic revival and collapse. For the case of frequency measurements, when the measured time intervals are larger, the atomic population inversion and survival probability of initial state decline rapidly at the early time and then both of them become unchanged. As the intervals become smaller, the curve of atomic population inversion and survival probability of initial state becomes more and more closed, and the decay of the atomic population inversion slows down. Especially, as the time intervals are sufficiently short (such as  $g\tau=0.001$ ), the QZE occurs, and the atomic population inversion and survival probability of initial state is close to unit.

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